

Agreements and disagreements between theories and experiments in nematoviscosity

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In this work a set of viscosity data selected from the nematic liquid crystals literature is compared with the currently accepted microscopic (molecular) theories for the nematic viscosity. It is shown that the kinetic theory of Doi [N. Kuzuu and M. Doi, *J. Phys. Soc. of Jpn.* **52**, 3486 (1983)] and the affine transformation theory of Hess [D. Baalss and S. Hess, *Phys. Rev. Lett.* **57**, 86 (1986)] equally predict that Miesowicz's coefficients of a given sample are not independent but, as it has been believed for many years [H. Knepe, F. Scheneider, and N. K. Sharma, *Ber. Bunsenges Phys. Chem.* **85**, 784 (1981)], they are connected by a linear relationship. Such conjecture gains a strong positive support when it is applied to a set of experimental data that we have collected. However, when these data are used to obtain the values of the parameters used to build these theories, it is found that the values assumed by them are in flagrant disagreement with the physical interpretation that they are supposed to have.

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I. INTRODUCTION

Since 1935, after the pioneer work of Miesowicz [1,2], it was established that in the presence of external fields some nematic liquid crystals (NLC) could present anisotropic viscosity, i.e., the value observed for their viscosity depends on the relative direction between an external field and the shearing plane. Nowadays, it is known that this phenomenon has its origin on the anisotropic shape of the nematic domains; as the direction of the external field is changed the collective mean orientation of these molecules also changes, establishing different physical conditions to the transport of momentum between adjacent shearing planes. During this period a large amount of theoretical and experimental researches was dedicated to the study of this phenomenon. All these investigations have been carried out by the Ericksen, Leslie, and Parodi (ELP) approach [3–8] that, without any detailed microscopic hypotheses, shows that the dissipation due to fluid flow on these anisotropic materials must be characterized by at least five different viscosity coefficients. So, the ELP approach gives rise to two complementary lines of research. First, it establishes the guides to the arduous work of measuring the viscosity coefficients of different nematic materials that, gradually, leads to the accumulation of a representative set of experimental data that, today, allows a systematic study of the nematic viscosity [9–17]. Second, the ELP approach also stimulates and orients an exhaustive theoretical work that has the microscopic (molecular) explanation of the nematic viscosity as objective [18–34].

In the past 20 years, two models have acquired relevance in search for a molecular explanation of the NLC viscosity: the kinetic model (KM) of Doi [21–24] and the affine connection model (ACM) of Hess [29–34]. The main aim of this work is to compare the predictions of these models with a set of experimental data selected from the NLC literature. It will be shown that, as it was conjectured many years ago [10], Miesowicz's coefficients seem to be correlated; regularities in the experimental data [27,28] suggest a linear connection between them. It will be also demonstrated that, from a formal point of view, such relationship seems to be ex-

plained by the current molecular theories quoted above. However, when these data are used to obtain the specific values of the parameters used in the construction of these theories, inconsistencies are found and it must be concluded that these parameters cannot have the physical meaning that was attributed to them.

II. FUNDAMENTALS

During the past 30 years the anisotropic viscosity of the NLC has been systematically investigated and, progressively, the values of the viscosity coefficients of some nematic compounds have been measured. In this work we will put together some sets of such measurements that, having the temperature as the control parameter, cover completely the interval ranging from nematic-isotropic to the nematic-crystalline phase transition points. With this procedure we avoid to arrive at conclusions that are not valid in the whole nematic phase, laying emphasis on a specific region of the nematic domain. Additionally, we have only selected viscosity data of those compounds for which the number of measured Leslie coefficients is enough to generate all Miesowicz's coefficients. Using these criteria we have chosen the following set of compounds; PAA (*p*-azoxyanisole), MBBA (*p'*-methoxybenzylidene-*p*-*n*-butylaniline), N4 (eutectic mixture of the 4-methoxy-4'-*n*-butylazoxybenzenes), EM (eutectic mixture of 4'-*n*-pentylphenyl 4-methoxybenzoate and 4'-*n*-pentylphenyl 4-*n*-hexyloxybenzoate), 5CBP (4-*n*-pentyl-4'-cyanobiphenyl), HBAB (*p*-*n*-hexyloxybenzylidene-*p'*-aminobenzonitrile), MIST (1:1:1-molar mixture of HBAB with *p*-*n*-butoxybenzylidene-*p'*-aminobenzonitrile and *p*-*n*-octanoyloxybenzylidene-*p'*-aminobenzonitrile). The authors who studied these measurements are quoted in Refs. [9–15,17]. For all these materials, the nematic phase exists in a different temperature interval. Consequently, the direct comparison of these data points can become difficult. The usual way to avoid this problem is to use a fixed point, habitually the nematic-isotropic (*N-I*) phase transition point, and obtain a rationalized temperature scale. But this procedure may give empha-

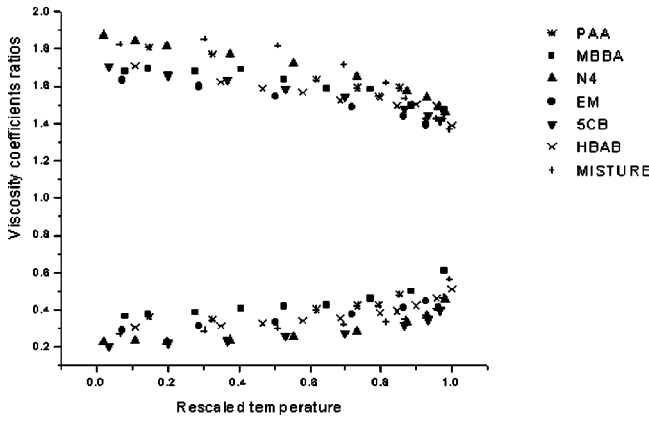


FIG. 1. Experimental data points from different sources have been collected and used to furnish the ratios between Miesowicz's coefficients η_3/η_2 and η_3/η_1 . The temperature scale is the nematic temperature scale as defined in the text. In all data, η_3/η_2 appears in the upper half of the picture, showing a regularity that suggests a universality. The data for η_3/η_1 appears at the lower half of the picture and also suggest a universality. The authors who studied these measurements are quoted in Refs. [9–15,17].

sis to the nematic-isotropic region. To prevent such incident, for each compound two fixed points have been chosen: the nematic-isotropic phase transition point, for which was attributed the temperature $t=1$, and the nematic-crystalline phase transition point, for which was attributed the temperature $t=0$. We will call such a temperature scale as the nematic temperature scale. In a previous work [28], we have used this temperature scale to construct curves of corresponding states that suggest that the viscosities of the above compounds obey universal relations. For this, the ratios between Miesowicz's coefficients η_3/η_1 and η_3/η_2 have been computed for each compound of this set. The results were collected in a unique graphic, as shown in Fig. 1. From this figure we see that, in the nematic temperature scale, the sets of points of different materials corresponding to η_3/η_1 and η_3/η_2 are not randomly distributed. The values of η_3/η_1 fluctuate around $\eta_3/\eta_1 \sim 0.4$, with a small increase with the rising temperature, being observed. The values of η_3/η_2 fluctuate around $\eta_3/\eta_2 \sim 1.7$, presenting a small decrease with the rising temperature. Furthermore, both sets of experimental data points seem to approach 1 as the temperature approaches the $N-I$ transition. Even though presenting significant fluctuations, it is clear that these experimental data points occupy two distinct regions. The regularity, agreement, and coherence observed on their distribution seem to be in accord with the idea of the existence of a physical rule connecting them; after a simple rescaling in the temperature, experimental data of different compounds, measured at different epochs by different researchers, are clearly distributed along two distinct regions, suggesting that they could coalesce along two single curves, expressing a universal relationship [27,28].

An important issue to be investigated concerns the ability of the current NLC rheologic theories to explain universal relations between Miesowicz's coefficients, such as that suggested by Fig. 1. The research on microscopic approaches for

the rheological properties of NLC has a long history that can go back to the times of Kirkwood and Auer [35]. Nevertheless, to our knowledge, the papers of Helfrich [36,37] and Martins [18,19] are the germs of our current understanding of this phenomenon. Presently, there are two different and representative approaches which are candidates to an explanation of the nematic viscosity. The kinetic model of Doi [20–24], starting from two constitutive equations (a microscopic model for the stresses tensor and a Fokker-Plank-like equation for an orientational distribution function of the director), was able to deduce a molecular expression for the Leslie coefficients. Concomitantly, Hess and co-workers [29–32] showed that through an affine transformation method the Leslie coefficients could be obtained. Such transformation can transform the spherical symmetric potential of a spherical molecule of a reference liquid in a potential with ellipsoidal symmetry of a uniaxial NLC. In this work we will compare the results of these theoretical models with experimental results. Therefore, we will not describe here the reasoning that led to these models, details can be found in Refs. [16,21–24,29–34].

III. AGREEMENTS BETWEEN THEORIES AND EXPERIMENTAL DATA

Let us start with the KM, which has produced some different molecular expressions for Miesowicz's coefficients, each one reflecting different kinds of approximations made during the calculations. Here, we will work with an expression by Kuzuu and Doi. Using the Leslie coefficients obtained by Kuzuu and Doi [24] it is easy to show that the corresponding Miesowicz's coefficients are given by

$$\begin{aligned} \eta_1 &= \frac{ck_B t}{4D_r} \frac{p^2-1}{p^2+1} \left\{ \frac{1}{35} \frac{p^2-1}{p^2+1} (14+5S_2+16S_4) \right. \\ &\quad \left. + S_2 \left(2 + \frac{1}{\lambda} \right) \right\}, \\ \eta_2 &= \frac{ck_B t}{4D_r} \frac{p^2-1}{p^2+1} \left\{ \frac{1}{35} \frac{p^2-1}{p^2+1} (14+5S_2+16S_4) \right. \\ &\quad \left. - S_2 \left(2 - \frac{1}{\lambda} \right) \right\}, \\ \eta_3 &= \frac{ck_B t}{4D_r} \left(\frac{p^2-1}{p^2+1} \right)^2 \left\{ \frac{2}{35} (7-5S_2-2S_4) \right\}, \end{aligned} \quad (1)$$

where c is the number of molecules per unit volume, p is the aspect ratio of the nematic molecule (ratio between its length and its width), D_r is the rotational diffusion, λ is the “tumbling parameter,” here assumed as constant [38], S_2 and S_4 are order parameters [39], and k_B , and t are the Boltzmann constant and temperature, in the nematic scale, respectively.

Likewise, the ACM expression for Miesowicz's coefficients is given by [32]

$$\begin{aligned}\eta_1 &= \eta^{ref} \left\{ 1 + S_2(p^2 - 1) + \frac{4}{15} \left(1 - \frac{10}{7} S_2 + \frac{3}{7} S_4 \right) (p - p^{-1})^2 \right\}, \\ \eta_2 &= \eta^{ref} \left\{ 1 + S_2(p^{-2} - 1) + \frac{4}{15} \left(1 - \frac{10}{7} S_2 + \frac{3}{7} S_4 \right) (p - p^{-1})^2 \right\}, \\ \eta_3 &= \eta^{ref} \left\{ 1 + \frac{1}{15} \left(4 - \frac{25}{7} S_2 - \frac{3}{7} S_4 \right) (p - p^{-1})^2 \right\},\end{aligned}\quad (2)$$

where η^{ref} is the reference viscosity of a liquid in which the molecules have spherical symmetry.

Observe that in both sets of equations, except for an overall coefficient that in the kinetic approach is given by $ck_B T(p^2 - 1)/[(p^2 + 1)4D_r]$ and in the affine approach is given by η^{ref} , the Miesowicz's coefficients are determined by S_2 and S_4 , which, in the nematic temperature scale, are universal parameters of the nematic phase [39]. Hence, expressed in this way, Miesowicz's coefficients can be determined by parameters of two different species: S_2 and S_4 , which are the same for all nematic compounds, and material dependent parameters that, like p , λ , D_r , and c for the KM and p and η^{ref} for the ACM, may change from compound to compound.

Equations (1) and (2) are not the most general expressions for Miesowicz's coefficients, which can be obtained with the KM or ACM. There are, for example, nonlinear equations in the order parameters S_2 and S_4 [40–42] that arise when the tumbling parameter λ is written in terms of the S_2 and S_4 . Nevertheless, as it has been demonstrated in Ref. [43], we would have $\lambda \approx 1$, and the inclusion of such nonlinearity will not change the results of our forthcoming conclusions [38]. Anyway, according to both theoretical approaches, Miesowicz's coefficients would have the general form

$$\eta_1 = \eta_1(\chi, S_2, S_4), \quad \eta_2 = \eta_2(\chi, S_2, S_4), \quad \eta_3 = \eta_3(\chi, S_2, S_4), \quad (3)$$

where we have introduced χ as a nonspecific symbol to represent the set of material dependent parameters of each model. We have here three equations with two universal parameters, S_2 and S_4 . These two parameters can be eliminated in these three equations and a unique equation, with a generic form

$$F(\eta_1, \eta_2, \eta_3, \chi) = 0, \quad (4)$$

is obtained. That is, as long as the three Miesowicz's coefficients are functions of two-order parameters S_2 and S_4 , it is proved that they are not independent; it is enough to know the material parameters χ and two of the Miesowicz's coefficients to determine the third.

An immediate application of the law given by Eq. (4) is the study of the form assumed by this law when the KM and ACM are used. As Eqs. (1) and (2) are linear in S_2 and S_4 , the form assumed by Eq. (4) in this case will be a linear relationship between Miesowicz's coefficients,

$$a_1 \eta_1 + a_2 \eta_2 + a_3 \eta_3 + a_4 = 0, \quad (5)$$

where the coefficients, a_i , $i = \{1, 2, 3, 4\}$, depend on the material parameters χ of each nematic specimen. Let us study this relation in detail. Using Eqs. (1) and (2) a straightforward calculation shows that

$$a_1^{KM} = 3 - \frac{2}{1+p^2} - \frac{1}{\lambda}, \quad a_1^{ACM} = 4 - \frac{5}{1+p^2},$$

$$a_2^{KM} = 1 + \frac{2}{1+p^2} + \frac{1}{\lambda}, \quad a_2^{ACM} = -1 + \frac{5}{1+p^2},$$

$$a_3^{KM} = 16, \quad a_3^{ACM} = 12,$$

$$a_4^{KM} = -\frac{2c(p^2 - 1)^2 k_B T}{(p^2 + 1)^2 D_r}, \quad a_4^{ACM} = -\frac{(4 + 7p^2 + 4p^4)}{p^2} \eta^{ref}. \quad (6)$$

Now the use of the symbols ACM and KM over coefficients, as in a_i^{ACM} and a_i^{KM} , indicate that these coefficients result from calculations made on the ACM and the KM respectively. According to the above equations, only the parameter a_4^{KM} seems to be temperature dependent. Nevertheless, if we remember that $D_r = k_B T / \xi_r$, where ξ_r is the rotational friction constant [44], we arrive at

$$a_4^{KM} = -\frac{2c(p^2 - 1)^2 \xi_r}{(p^2 + 1)^2}, \quad (7)$$

which shows that the sets of coefficients of Eq. (6) are indeed temperature independent. So, if a set of Miesowicz's coefficients resulting from the measurement of a unique sample at different temperatures are distributed in a three-dimensional space, where each axis represents one of the Miesowicz's coefficients, the points representing the results of such measurements would be entirely contained by a plane with orientation given by $\vec{a} = (a_1, a_2, a_3)$ and a linear coefficient given by a_4 . From we will call this plane the Miesowicz plane.

Some years ago, Knepe, Scheneider, and Sharma (KSS) [10] found that the measured Miesowicz's coefficients of some compounds seem to be connected. More recently, Janik *et al.* [16] found the same kind of relationship between their experimental data. Apparently, the result expressed through Eq. (5) is the theoretical justification of such achievements. For example, using their experimental data KSS had found that η_2/η_1 and η_3/η_1 seem to be linearly related. According to Eq. (5), one has

$$\frac{\eta_2}{\eta_1} = -\frac{a_3}{a_2} \frac{\eta_3}{\eta_1} - \frac{a_1}{a_2} - \frac{a_4}{a_2} \frac{1}{\eta_1}. \quad (8)$$

Consequently, the KSS discovery would be an approximation to Eq. (5), whose degree of approximation would depend on the values assumed by the last term of the above equation. Namely, as η_1 is the greatest Miesowicz's coefficient, it can be assumed that $[a_4/(\eta_1 a_2)] \approx 0$, leading to

$$\frac{\eta_2}{\eta_1} = \alpha \frac{\eta_3}{\eta_1} + \beta, \quad (9)$$

which is the KSS relation, where $\alpha = -a_3/a_2$, $\beta = -a_1/a_2$, and

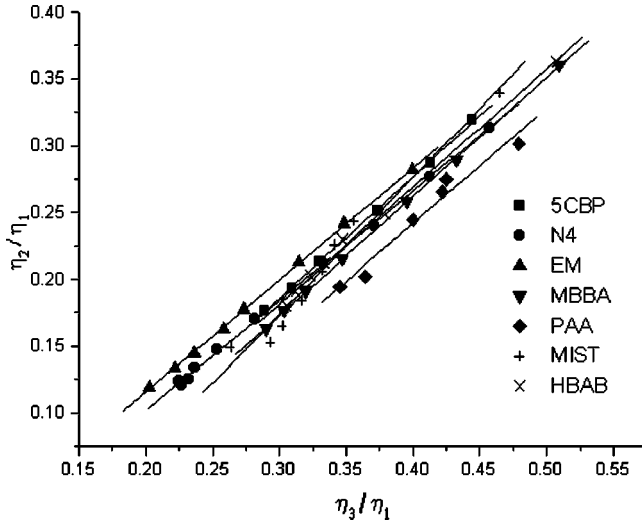


FIG. 2. In this figure the set of experimental data point collected from the NLC literature has been used to compute a graph of η_2/η_1 versus η_3/η_1 . As it was observed in Ref. [10], the experimental data points of all compounds suggest that these points would be distributed according to a straight line and, furthermore, the angular coefficient of all these lines seems to assume approximately the same value.

$$\alpha^{KM} = -\frac{16(p^2+1)\lambda}{1+3\lambda+(\lambda+1)p^2}, \quad \alpha^{ACM} = \frac{12(1+p^2)}{-4+p^2},$$

$$\beta^{KM} = \frac{1-\lambda+(1-3\lambda)p^2}{1+3\lambda+(\lambda+1)p^2}, \quad \beta^{ACM} = \frac{-1+4p^2}{-4+p^2}. \quad (10)$$

Figure 2 shows a superposition of the curves exhibiting this relation for each compound of our set. These graphs seem to confirm that the experimental data really gives a linear relationship between η_3/η_1 and η_2/η_1 .

As the set of compounds used in this paper is larger than the set used by KSS, and Eq. (9) is an approximation to Eq. (5), we must investigate if our set of Miesowicz's coefficients is linked by a relationship such as Eq. (5). Let us give a positive answer to this question by examining Eq. (5) a bit further. According to it, Miesowicz's coefficients would be distributed over a plane, namely, the Miesowicz plane. But, once over this plane, are the points representing the measurement of Miesowicz's coefficients randomly distributed or are they organized according to some physical law? It is straightforward to show that if the following change of variables are made:

$$\chi_1 = \eta_1 - \eta_2 \quad \text{and} \quad \chi_2 = \eta_1 + \eta_2 + 8\eta_3, \quad (11)$$

Eq. (5) acquires the form

$$\chi_2 = a_0 + a_1\chi_1, \quad (12)$$

where

$$a_0^{KM} = \frac{c(p^2-1)^2}{(p^2+1)^2} \xi_r, \quad a_0^{ACM} = \frac{2(4+7p^2+4p^4)}{3p^2},$$

$$a_1^{KM} = \frac{1-(\lambda-1)p^2+\lambda}{2(p^2+1)\lambda}, \quad a_1^{ACM} = -\frac{5(-1+p^2)}{3(1+p^2)}. \quad (13)$$

Consequently, both theoretical models predict that on the Miesowicz plane the experimental points would be distributed according to the straight line given by Eq. (11). We have subjected the data of all compounds that we collected in the literature to the transformation given by Eq. (11) and plotted the results in a graph, as shown in the Fig. 3. It is clear that for each compound the curve of χ_2 as a function of χ_1 is very well approximated by a straight line, which confirms that Miesowicz's coefficients are not independent and seem to give a strong experimental support to the above mentioned theoretical models. In Table I, the adjusted values for a_0 and a_1 for each compound are shown.

To conclude this section, in Fig. 4 all data points of Fig. 3 are superimposed. Even though recognizing that the parameters of Eq. (12), a_0 and a_1 , are compound dependent, it is clear that this figure indicates that among the nematic compounds these parameters do not present large variations. If this is so, the next obvious step would be the determination of the values of these parameters with the use of the experimental data points. In the following section, it will be shown that the values assumed by these parameters do not give physical consistency for the theories in which they are defined.

IV. DISAGREEMENTS BETWEEN THEORIES AND EXPERIMENTAL DATA

According to the theories developed above, it seems clear that Miesowicz's coefficients would be connected. Furthermore, the experimental data seems to give support to this conclusion, indicating that such a connection would be linear. Here, we will use the experimental data to compute the values of the material parameters used in each of the models introduced above. Let us begin with the ACM. According to it, Eq. (13), the angular coefficient a_1^{ACM} of the straight line in the Miesowicz plane would be given by $a_1^{ACM} = -5(-1+p^2)/[3(1+p^2)]$. An immediate analysis of this equation for nematic compounds ($p^2 > 1$) show that it would give a negative value for a_1^{ACM} . Figures 3 and 4 clearly show that, for all compounds, the experimental data predict that this coefficient is positive. Consequently, there is an evident disagreement between the ACM and these data. So, at least from the point of view of the values that must be attributed to these parameters, this theory is in flagrant disagreement with the experimental results.

Let us make the same analysis for the KM. According to Eq. (13), the angular coefficient of the straight line in the Miesowicz plane would be given by $a_1^{KM} = [1-(\lambda-1)p^2+\lambda]/[2(p^2+1)\lambda]$, which depends on p and λ . From this equation it follows that

$$p^2 = -\frac{1+\lambda(1-2a_1^{KM})}{1-2a_1^{KM}-\lambda}.$$

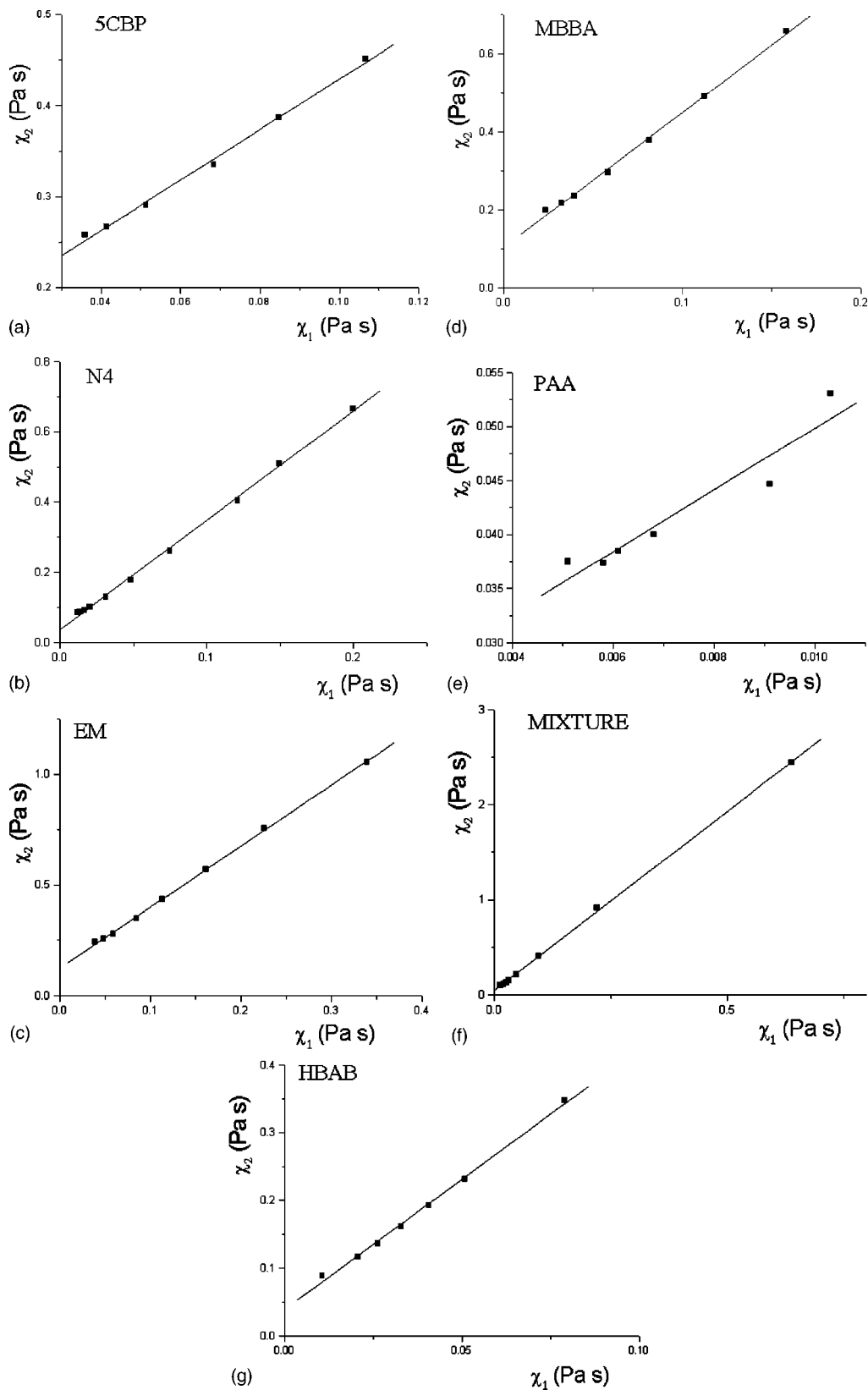


FIG. 3. In this set of figures the graphs of $\chi_2 = \eta_1 + \eta_2 + 8\eta_3$ are shown as a function of $\chi_1 = \eta_1 - \eta_2$ for the set of experimental data point that have been collected in the liquid crystals literature, where η_1 , η_2 , and η_3 are Miesowicz's coefficients, as defined in Ref. [28]. The authors who studied each of these measurements are given in the paper and in the references. The corresponding compound names are shown in each figure and detailed in the text of the paper. There is an astonishing regularity in the distribution of these data points, which confirm the theoretical prediction of a linear relationship between χ_1 and χ_2 .

TABLE I. In this table, the values of linear and angular coefficients of the straight line that best adjust the data points of Fig. 3 are shown. The mean values shown at the bottom of the table are the ones used to plot the straight line of Fig. 4.

Compound	a_0	a_1
CBP5	0.1517	2.77422
N4	0.03712	3.1193
EM	0.12388	2.75958
MBBA	0.10352	3.46126
PAA	0.02123	2.86641
MIST	0.04314	3.77719
HBAB	0.03935	3.84565
Average value	0.0742771	3.2290871

So, it is straightforward to show that the condition $p^2 > 1$ only has a solution if λ and a_1^K satisfy the relations

$$a_1^{KM} > \frac{1-\lambda}{2} \quad \text{and} \quad a_1^{KM} < \frac{1}{1+\lambda}.$$

Due to the physical meaning of λ [39], it is clear that we would have $\lambda > 0$ (indeed, as it was shown in detail in Ref. [43], one would expect that $\lambda \approx 1$). Consequently, while the first of these inequalities does not impose important restrictions on the values of the parameter a_1^{KM} , the second inequality shows that this version of the kinetic model and the experimental data have serious disagreements. The experimental data show that one would expect that $a_1^{KM} \geq 2.5$, while, according to the second of the above inequalities, such

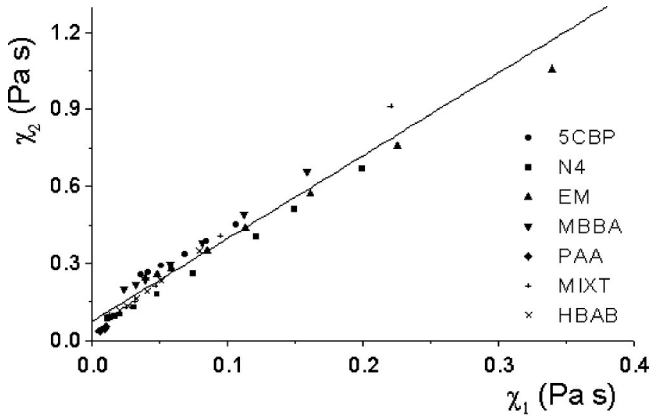


FIG. 4. In this figure the curves that have been separately displayed in Fig. 3 are joined together. As it was shown in detail along the paper, the linear and angular coefficients of each straight line should be fixed by the particular parameters of each nematic compound. Nevertheless, the regular distribution of points in this figure reveals that among the diversity of the nematic compounds these parameters cannot have a substantial change. Of course, as it is shown in Table I, due to the particular character of each compound there are variations in these parameters. But, as it is evident from the figure, the fact that all points are consistently distributed along the same mean straight line is a consequence of the similarity of the parameters of the nematic phase.

a value of a_1^{KM} would imply a negative value for λ , ($\lambda < -3/5$), which is absurd. Furthermore, the KM predicts that the values of the angular coefficient of the curve given in Fig. 2 would be given by $\alpha^{KM} = -16(p^2 + 1)\lambda/[1 + 3\lambda + (\lambda + 1)p^2]$, which is negative for all values p and λ , being in disagreement with the results obtained with the experimental data.

V. FINAL REMARKS AND CONCLUSION

According to the analysis developed above, it has been shown that the two main theories developed to explain the viscosity of the NLC predict that Miesowicz's coefficients are not independent but are linearly connected. A representative set of experimental data collected from the liquid crystal literature gives a strong confirmation that Miesowicz's coefficients really obey such a relationship. Nevertheless, when these experimental data are used to predict the values of the parameters defined by these theories, it is found that the values obtained by them are in disagreement with the central interpretation that these theories give to them; they assume nonphysical values.

As one cannot doubt the correctness of the experimental data, it seems clear that the linear relationship between $\chi_1 = \eta_1 - \eta_2$ and $\chi_2 = \eta_1 + \eta_2 + 8\eta_3$ exhibited in Fig. 3, and its congener given by Eq. (9), exists. Furthermore, the fact that the microscopic theories developed from completely different starting points give the same general form of these curves seems to indicate that the fundamentals of these theories must be seriously considered. Otherwise, the fact that, when confronted with experimental data, both theories equally fail, giving an inconsistent interpretation of their parameters, suggests that some essential element is missing on these fundamentals. Of course, it can be affirmed that in our analysis we have not used the most general expressions produced by these theories existing in both approaches other expressions for the Miesowicz's coefficients that take into account more detailed approximations [22]. About the generality of these variants, see the work of Chrzanowska [38], which establishes that essentially all these approaches are equivalent. Anyway, we have looked into the liquid crystals literature and found that the majority of these expressions have the form [22,25]

$$\begin{aligned} \eta_1 &= \rho_1 + \frac{1}{2}(A+B)S_2 + 4CS_4, \\ \eta_2 &= \rho_2 + \frac{1}{2}(A-B)S_2 + 4CS_4, \end{aligned} \quad (14)$$

$$\eta_3 = \rho_3 + DS_2 - CS_4,$$

where ρ_1 , ρ_2 , ρ_3 , A , B , C , and D are constants. Equations (14) have Eqs. (1) and (2) as particular cases. It is a straightforward exercise to show that for all theories having the form of Eqs. (14), one would have

$$\begin{aligned} \chi_2 &= \frac{-(A-B+8D)\rho_1 + (A+B+8D)\rho_2 + 8B\rho_3}{B} \\ &+ \frac{(A+8D)}{B} \chi_1. \end{aligned} \quad (15)$$

Consequently, for all these theories we would have

$$a_1 = \frac{(A + 8D)}{B},$$

which shows that the angular coefficient of the curve of the Miesowicz plane must be determined by the parameters A , B , and D of Eq. (14). Furthermore, as B is easily determined by the relation $\eta_1 - \eta_2 = BS_2$, we conclude that if one wants to look for the origin of the disagreements found in the theories exposed above, the significance of the coefficients A and D must be investigated.

An equivalent result can be obtained if Eq. (9) is used. Equation (14) gives the angular coefficient α of that equation as

$$\alpha = -\frac{8B}{A + B + 8D}.$$

Consequently, as above, this relation indicates that A , B , and D are the parameters whose significance must be investigated.

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